Lösung Aufgabe 4c

Cholesky Zerlegung:

$$\boldsymbol{A} = \begin{pmatrix} 4 & 2 & 4 & 4 \\ 2 & 10 & 17 & 11 \\ 4 & 17 & 33 & 29 \\ 4 & 11 & 29 & 39 \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} & l_{41} \\ 0 & l_{22} & l_{32} & l_{42} \\ 0 & 0 & l_{33} & l_{43} \\ 0 & 0 & 0 & l_{44} \end{pmatrix} = \boldsymbol{L}\boldsymbol{L}^T$$

Praktische Berechnung der Cholesky-Zerlegung:

$$l_{ii} = (a_{ii} - \sum_{k=1}^{i-1} {l_{ik}}^2)^{\frac{1}{2}}$$

$$l_{ji} = \frac{1}{l_{ii}}(a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik})$$

Lösung:

$$l_{11} = (a_{11} - \sum_{k=1}^{1-1} {l_{1k}}^2)^{\frac{1}{2}} = (4-0)^{\frac{1}{2}} = 2$$

$$l_{21} = \frac{1}{l_{11}}(a_{21} - \sum_{k=1}^{1-1} l_{2k} l_{1k}) = \frac{a_{21}}{l_{11}} = \frac{2}{2} = 1$$

$$l_{31} = \frac{1}{l_{11}}(a_{31} - \sum_{k=1}^{1-1} l_{3k} l_{1k}) = \frac{a_{31}}{l_{11}} = \frac{4}{2} = 2$$

$$l_{41} = \frac{1}{l_{11}}(a_{41} - \sum_{k=1}^{1-1} l_{4k} l_{1k}) = \frac{a_{41}}{l_{11}} = \frac{4}{2} = 2$$

$$l_{22} = (a_{22} - \sum_{k=1}^{2-1} {l_{2k}}^2)^{\frac{1}{2}} = (10 - 1^2)^{\frac{1}{2}} = 3$$

$$l_{32} = \frac{1}{l_{22}}(a_{32} - \sum_{k=1}^{2-1} l_{3k} l_{2k}) = \frac{1}{3}(17 - 2 \cdot 1) = 5$$

$$l_{42} = \frac{1}{l_{22}}(a_{42} - \sum_{k=1}^{2-1} l_{4k} l_{2k}) = \frac{1}{3}(11 - 2 \cdot 1) = 3$$

$$l_{33} = (a_{33} - \sum_{k=1}^{3-1} {l_{3k}}^2)^{\frac{1}{2}} = (a_{33} - ({l_{31}}^2 + {l_{32}}^2)^{\frac{1}{2}} = (33 - 2^2 - 5^2)^{\frac{1}{2}} = 2$$

$$l_{43} = \frac{1}{l_{33}}(a_{43} - \sum_{k=1}^{3-1} l_{4k} l_{3k}) = \frac{1}{2}(29 - (2 \cdot 2 + 3 \cdot 5)) = 5$$

$$l_{44} = (a_{44} - \sum_{k=1}^{4-1} {l_{4k}}^2)^{\frac{1}{2}} = (39 - (2^2 + 3^2 + 5^2))^{\frac{1}{2}} = 1$$

$$\Rightarrow \boldsymbol{L} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 2 & 3 & 5 & 1 \end{pmatrix}$$