Herleitung der Kovarianzmatrizen

$$
\begin{aligned}
\begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{b}} \end{pmatrix} &= (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{y} \\
\Rightarrow \begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{b}} - \boldsymbol{b} \end{pmatrix} &= (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}\Bigg[\boldsymbol{C}'\boldsymbol{R}^{-1}\{\boldsymbol{C}\begin{pmatrix} \boldsymbol{\beta} \\ \boldsymbol{b} \end{pmatrix} + \boldsymbol{\varepsilon}\} \\
&\qquad -(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{pmatrix}\Bigg] \\
\Rightarrow \operatorname{Cov}\begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{b}} - \boldsymbol{b} \end{pmatrix} &= \operatorname{Cov}\left[(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}\left\{\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{\varepsilon} - \boldsymbol{A}\begin{pmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{pmatrix}\right\}\right] \\
&= (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A}\boldsymbol{A}^{-1}\boldsymbol{A}) \cdot \\
&\qquad (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1} \\
&= (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}.
\end{aligned}
$$

$$
\begin{aligned}
\begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{b}} \end{pmatrix} &= (\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A})^{-1}\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{y} \\
\Rightarrow \operatorname{Cov}\left(\left. \begin{pmatrix} \widehat{\boldsymbol{\beta}} \\ \widehat{\boldsymbol{b}} \end{pmatrix} \right| \boldsymbol{b} \right) &= \left(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A}\right)^{-1} \boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{R}\boldsymbol{R}^{-1}\boldsymbol{C} \left(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A}\right)^{-1} \\
&= \left(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A}\right)^{-1} \boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} \left(\boldsymbol{C}'\boldsymbol{R}^{-1}\boldsymbol{C} + \boldsymbol{A}\right)^{-1}.
\end{aligned}
$$