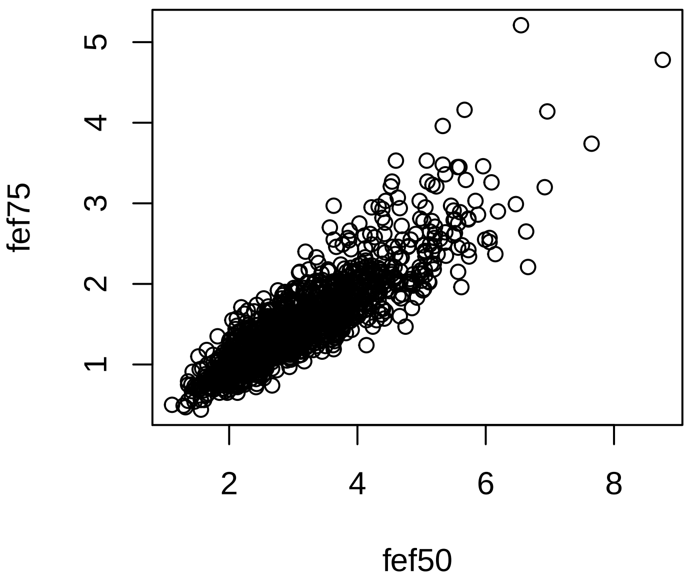
Aufgabe 1

Einlesen der Daten

```
load("atemwege.rda")
attach(atem)
```

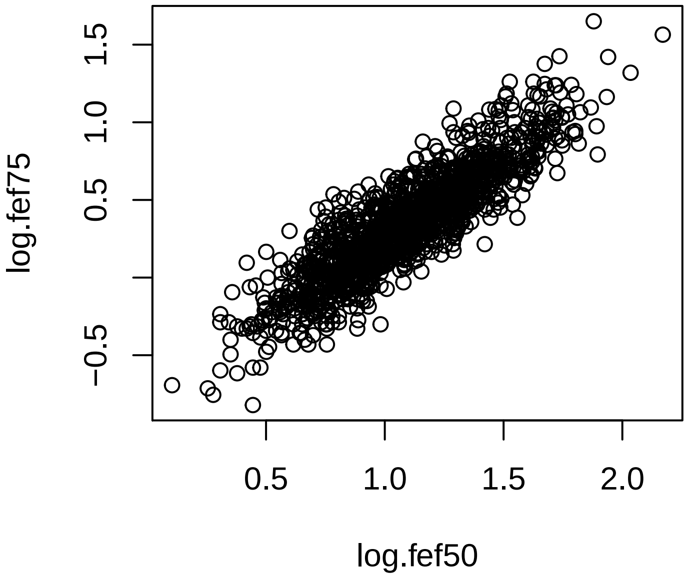
Vorab-Analyse

```
plot(fef50,fef75)
```




```
# die marginalen Verteilung erscheinen eher linkssteil,
# daher betrachten wir die logarithmierten Variablen
log.fef50<-log(fef50)
log.fef75<-log(fef75)

plot(log.fef50,log.fef75)
```




```
cor(log.fef50,log.fef75)

## [1] 0.8846577

# betr?chtliche Korrelation (0.8846577) zwischen den Variablen
```

a) Multivariater Test fuer Erwartungswert im Ein-Stichproben-Fall (Kovarianz unbekannt)

```
n <- nrow(atem)
p <- 2
alpha <- 0.01
x.quer <- c(mean(log.fef50),mean(log.fef75))
mu0    <- c(log(3),log(1.4))
S      <- var(cbind(log.fef50,log.fef75))

# Teststatistik:
T.s <- n*(n-p)/(p*(n-1))*(x.quer-mu0)%*%solve(S) %*%(x.quer-mu0)
T.s

##          [,1]
## [1,] 11.99079

qf(1-alpha,p,n-p)

## [1] 4.621201

# p-Wert
pf(T.s,p,n-p,lower.tail=F)

##              [,1]
## [1,] 6.902319e-06
```

$\Rightarrow$ Mittelwertsvektor signifikant verschieden von mu0!

Funktion zur Berechnung der Mahalanobis-Distanz D^2 (Ellipse)

```
d2.function <-function(mu1,mu2,S,xquer){

  Sinv <- solve(S)
  res <-  Sinv[1,1]*(xquer[1]-mu1)^2+
          (Sinv[1,2]+ Sinv[2,1])*(xquer[2]-mu2)*(xquer[1]-mu1)+
           Sinv[2,2]*(xquer[2]-mu2)^2
  return(res)
}
```

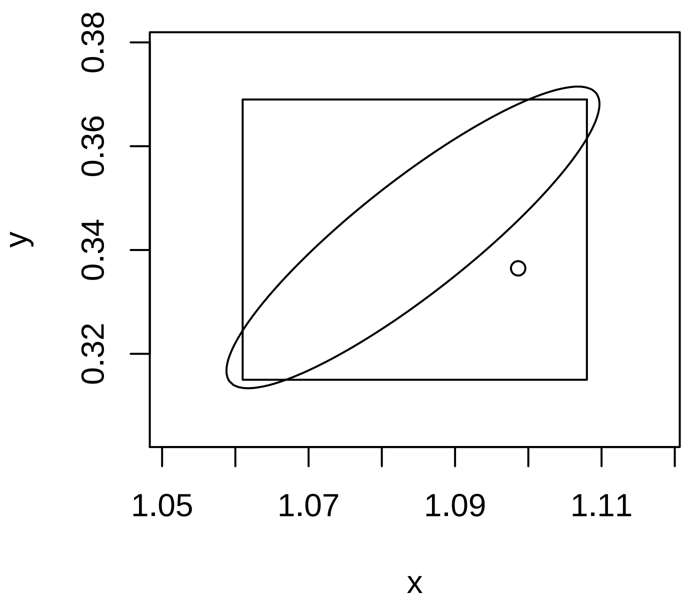
```
# Bonferroni-Rechteck:
x <- c(1.108,1.108,1.061,1.061,1.108)
y <- c(0.369,0.315,0.315,0.369,0.369)
c <- 0.01
plot(x,y,
     type="l",xlab="x",ylab="y",xlim=c(1.061-c,1.108+c),
     ylim=c(0.315-c,0.369+c))

#Erstellen eines Grids f?r die Konfidenzellipsoide
x.seq<-seq(1.061-c,1.108+c,length=100)
y.seq<-seq(0.315-c,0.369+c,length=100)

#Berechnung des ?u?eren Produkts von x.seq und y.seq, Berechnung der
#Mahalanobis-Distanz mit diesen Werten
z<-outer(x.seq,y.seq,d2.function,S=S,xquer=x.quer)

#Einzeichnen der Ellipse
ellipse<-contourLines(x.seq,y.seq,z,levels=(n-1)*p/((n-p)*n)*qf(1-alpha,p,n-p))
lines(ellipse[[1]]$x,ellipse[[1]]$y)

points(mu0[1],mu0[2])
```



b) Multivariater Test fuer die Symmetrie im Ein-Stichprobenfall

```
log.pef <-log(pef)

y     <- cbind(log.pef-log.fef50,log.pef-log.fef75)
yquer <- c(mean(y[,1]),mean(y[,2]))
S     <- var(y)

# Teststatistik
T.s  <-(n-p)*n/(p*(n-1))*(yquer)%*%solve(S) %*%(yquer)
T.s
```

```
##          [,1]
## [1,] 16718.75
```

```
qf(1-alpha,p,n-p)
```

```
## [1] 4.621201
```

```
# p-Wert:
pf(T.s,p,n-p,lower.tail=F)
```

```
##      [,1]
## [1,]    0
```

⇒ H0 wird abgelehnt!

c) Multivariater Test im Zwei-Stichproben-Fall fuer unabhaengige Stichproben.

```
varmat <- cbind(log.fef50,log.fef75)

x1.quer <- apply(varmat[sex==1,],2,mean)
x2.quer <- apply(varmat[sex==2,],2,mean)

#Berechnung Stichprobenumfaenge in den Gruppen
n1<- table(sex)[1]
n2<- table(sex)[2]

#Berechnung der Kovarianzmatrizen in den Gruppen
S1<-var(varmat[sex==1,])
S2<-var(varmat[sex==2,])

#Berechnung der gepoolten Kovarianzmatrix
S<-1/(n1+n2-2)*((n1-1)*S1+(n2-1)*S2)
print(S)
```

```
##            log.fef50  log.fef75
## log.fef50 0.09288881 0.09399175
## log.fef75 0.09399175 0.12135721
```

```
print(var(varmat))
```

```
##            log.fef50  log.fef75
## log.fef50 0.09318615 0.09406287
## log.fef75 0.09406287 0.12132062
```

```
#Teststatistik T^2 (vgl. Aufgabe 1)
T2<-n1*n2/(n1+n2)*(x1.quer-x2.quer)%*%solve(S)%*%(x1.quer-x2.quer)
T2
```

```
##           [,1]
## [1,] 12.51178

# p-Wert
pwert<-pf((n1+n2-p-1)/((n1+n2-2)*p)*T2,p,n1+n2-p-1,lower.tail=F)
pwert

##              [,1]
## [1,] 0.001985544
```

⇒ H0 wird abgelehnt!

Univariate Tests

```
t.test(log.fef50[sex==1],log.fef50[sex==2],var.equal=T)

##
##  Two Sample t-test
##
## data:  log.fef50[sex == 1] and log.fef50[sex == 2]
## t = 2.2908, df = 1326, p-value = 0.02213
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.005516469 0.071295426
## sample estimates:
## mean of x mean of y
##  1.102153  1.063747

t.test(log.fef75[sex==1],log.fef75[sex==2],var.equal=T)

##
##  Two Sample t-test
##
## data:  log.fef75[sex == 1] and log.fef75[sex == 2]
## t = 0.7745, df = 1326, p-value = 0.4388
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.02275148  0.05243470
## sample estimates:
## mean of x mean of y
## 0.3493448 0.3345032
```

⇒ Univariate Tests kommen hier zu unterschiedlichen Ergebnissen!