

This exercise sheet will introduce you to the estimation of **Linear Mixed Models**. The exercises refer to the content of the third lecture slides.

Exercise 1:

In this exercise, we are working with the data set `rats` once more (see sheet 1, exercise 1). Use the code available on the homepage to import `rats.long.RData` in R as well as to prepare the data set for the following analysis.

- a) Estimate a linear mixed model with a linear slope in `logT` and subject-specific random intercepts using the function `lme()`.
 - i) Formulate the underlying model for the response vector \mathbf{Y}_i of the i -th rat and specify the dimensions of all components.
 - ii) What is the estimated marginal correlation between two measurements on the same rat?
Note: The function `getVarCov()` might be helpful.
 - iii) What is the estimated conditional correlation between two measurements on the same rat?
- b) In order to check graphically whether subject-specific slopes would be useful as well, estimate a separate linear model for each rat with at least 3 measurements using the function `lmList()` and plot the estimators and the confidence intervals for the intercept and for `logT` using the function `plot(intervals())`.

Note: The data set has to be a `groupedData` object with grouping variable `SUBJECT`.

- i) Why are separate linear models for the single rats only suitable for such an illustration?
- ii) As a fairly large variation can also be seen in the estimates of `logT`, fit a linear mixed model with a linear slope in `logT` and with subject-specific random intercepts and slopes. For better comparability only use rats with at least 3 measurements.
- iii) Determine the estimated covariance matrix $\hat{\mathbf{D}}$ of the random effects. What is the estimated correlation between the random intercepts and slopes?
- iv) Now compare the coefficient estimates and the fitted values of the subject-specific linear models and of the linear mixed model from ii) using `plot(compareFits())` and `plot(comparePred())`. Describe what you notice. What is the name of the observed effect and how can it be explained?