This exercise sheet concentrates on the concrete estimation of linear mixed and marginal models with different methods and under different assumptions for the correlation structure. The exercises refer to the content of the fourth lecture slides.

Exercise 1:
In this exercise, we are working with the orthodontic growth data from the data set `Orthodont` included in the package `nlme`, which is already in `groupedData` format. The data set contains measurements of jaw sizes of 27 boys and girls aged 8 to 14 years.

a) Familiarize yourself with the data and their grouping structure at first. For this purpose use `?Orthodont`, `str(Orthodont)` and `getGroups(Orthodont)`.

We consider the random intercept model `m_RI`

$$Y_{ij} = \beta_0 + \beta_1 \texttt{age}_{ij} + \beta_2 \texttt{Sex}_i + b_{0i} + \varepsilon_{ij}.$$

b) The function `lme()` provides two estimation methods "ML" and "REML" (default). Explain and compare the two methods briefly (without formulating the likelihoods exlicitly).

c) Estimate the model `m_RI` with both methods and compare them with respect to the estimated variance and resulting predictions of the random intercepts.

Now we discuss marginal models under different assumptions for the correlation structure. Such marginal models can be estimated with the function `gls()` (**G**eneralized **L**east **S**quares) in the `R` package `nlme`.

d) Estimate a model with the same fixed effects as before and with REML estimation. Assume that measurements between subjects are independent and specify an unstructured correlation structure for measurements within a subject.

e) Compare the estimated correlation matrix with the correlation matrix, which results from the marginal approach of the model `m_RI`. Interpret the result.

f) Now estimate a model with simplified correlation structure which corresponds to the marginal correlation structure of the model `m_RI`.

g) Is the model suggested in f) equivalent to `m_RI`? Give reasons for your answer.